BAC Matematica 2025 – examen – M3

Subiectul I

Exercitiul 1

$$\frac{1}{10}+3\left(\frac{1}{2}-\frac{1}{5}\right)=\frac{1}{10}+3\cdot\frac{3}{10}=\frac{1}{10}+\frac{9}{10}=1.$$

Exercitiul 2

$$f(2)=a+f(0)\Leftrightarrow 12-3=a-3\Leftrightarrow a=12.$$

Exercitiul 3

$$5^{2x}=5^{3-x}\Leftrightarrow 2x=3-x\Leftrightarrow x=1.$$

Asadar, $S=\{1\}$.

Exercitiul 4

$$6n>25\Leftrightarrow n>\frac{25}{6}.$$

Din cele zece cifre, $5,6,7,8,9$ verifica inegalitatea. Sunt cinci cazuri favorabile din zece posibile, deci $P=\dfrac{5}{10}=\dfrac{1}{2}$.

Exercitiul 5

$$AB=\sqrt{(x_B-x_A)^2+(y_B-y_A)^2}=\sqrt{(2-2)^2+(1-0)^2}=1,$$
$$BC=\sqrt{(x_C-x_B)^2+(y_C-y_B)^2}=\sqrt{(6-2)^2+(4-1)^2}=5.$$

Astfel, $BC=a\cdot AB\Leftrightarrow 5=a\cdot 1\Leftrightarrow a=5$.

Exercitiul 6

$$MP=\frac{MN}{4}=2,\qquad \mathcal{A}_{MNP}=\frac{MN\cdot MP}{2}=\frac{8\cdot 2}{2}=8.$$

Subiectul II

Exercitiul 1

a)

$$\det A = \begin{vmatrix} 5 & 2 \\ 2 & 1 \end{vmatrix} = 5 \cdot 1 - 2 \cdot 2 = 1.$$

b)

$$3A - 5I_2 = \begin{pmatrix} 15 & 6 \\ 6 & 3 \end{pmatrix} - \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix} = \begin{pmatrix} 10 & 6 \\ 6 & -2 \end{pmatrix} = 2B.$$

Cum $B \neq O_2$, $2B = xB \Leftrightarrow x = 2$.

c)

$$A(B - A) = \begin{pmatrix} 5 & 2 \\ 2 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & -2 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix}.$$

$$\det\big(A(B - A) + xI_2\big) \leq 2 \Leftrightarrow \begin{vmatrix} x+2 & 1 \\ 1 & x \end{vmatrix} \leq 2$$
$$\Leftrightarrow x(x+2) - 1 \leq 2 \Leftrightarrow (x+3)(x-1) \leq 0.$$

Produsul este mai mic sau egal cu zero intre radacinile -3 si 1, inclusiv la capete. Multimea ceruta este $[-3, 1]$.

Exercitiul 2

a)

$$f(1) = 1 - 3 + 1 + 1 = 0.$$

b) Relatiile lui Viete dau $x_1 + x_2 + x_3 = 3$ si $x_1x_2x_3 = -m$. Conditia ceruta devine

$$2(x_1 + x_2 + x_3) = 1 + x_1x_2x_3 \Leftrightarrow 6 = 1 - m \Leftrightarrow m = -5.$$

c) Teorema restului da $f(2) = -5$, deci

$$8 - 12 + 2 + m = -5 \Leftrightarrow m = -3.$$

$$\begin{array}{rr|l} X^3 - 3X^2 + X - 3 & & X^2 + 1 \\ \cline{3-3} -X^3 \qquad\quad - X & & X - 3 \\ \cline{1-2} -3X^2 \qquad - 3 & & \\ +3X^2 \qquad + 3 & & \\ \cline{1-2} 0 & & \end{array}$$

Catul este $q = X - 3$, iar restul este $r = 0$. Asadar, $f = (X^2 + 1)(X - 3)$, deci $X^2 + 1$ divide f.

Subiectul III

Exercitiul 1

a)

$$f'(x) = \left(\frac{x-2}{e^x}\right)' = \frac{e^x - (x-2)e^x}{e^{2x}} = \frac{3-x}{e^x}, \quad \forall x \in \mathbb{R}.$$

b)

$$f(0) = -2, \qquad f'(0) = 3.$$

Tangenta are ecuatia $y - f(x_0) = f'(x_0)(x - x_0)$, deci

$$y + 2 = 3(x - 0) \Leftrightarrow y = 3x - 2.$$

c) Cum $e^x > 0$, $f'(x) > 0$ pentru $1 \leq x < 3$ si $f'(x) < 0$ pentru $3 < x \leq 4$.

x	1		3		4
$f'(x)$		$+$	0	$-$	
$f(x)$	$-\frac{1}{e}$	$\nearrow$	$\frac{1}{e^3}$	$\searrow$	$\frac{2}{e^4}$

Deoarece $f(1) < 0 < f(4)$, minimul este $f(1)$, iar maximul este $f(3)$. Inmultind cu $e^x > 0$, obtinem

$$-\frac{1}{e} \leq \frac{x-2}{e^x} \leq \frac{1}{e^3} \Rightarrow -e^{x-1} \leq x - 2 \leq e^{x-3}, \quad \forall x \in [1, 4].$$

Exercitiul 2

a)

$$\int_0^1 (f(x) - x^2 - 8)\, dx = \int_0^1 4x\, dx = 2x^2\Big|_0^1 = 2.$$

b)

$$\int_0^8 \frac{x}{f(x) - 4x}\, dx = \int_0^8 \frac{x}{x^2 + 8}\, dx.$$

Cu substitutia $t = x^2 + 8$, $dt = 2x\, dx$, capetele devin $t = 8$ pentru $x = 0$ si $t = 72$ pentru $x = 8$. Atunci

$$\int_0^8 \frac{x}{x^2 + 8}\, dx = \frac{1}{2}\int_8^{72} \frac{1}{t}\, dt = \frac{1}{2}\ln t\Big|_8^{72} = \frac{1}{2}\ln 9 = \ln 3.$$

c)

$$\int_0^a \frac{1}{f(x) - 4}\, dx = \int_0^a \frac{1}{(x+2)^2}\, dx.$$

Cu substitutia $t = x + 2$, $dt = dx$, capetele devin 2 si $a + 2$. Cum $a > 0$, numitorul nu se anuleaza. Astfel,

$$\int_0^a \frac{1}{(x+2)^2}\, dx = \int_2^{a+2} t^{-2}\, dt = -\frac{1}{t}\Big|_2^{a+2} = \frac{1}{2} - \frac{1}{a+2}.$$

Conditia este $\frac{1}{2} - \frac{1}{a+2} = \frac{1}{4}$, deci $a + 2 = 4$, de unde $a = 2 > 0$, valoare care convine.